ST.ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY

(An ISO 9001:2015 Certified Institution)

Anguchettypalayam, Panruti – 607106.

QUESTION BANK

PERIOD: JULY - NOV 2018 **BATCH**: 2015 – 2019

BRANCH: ECE **YEAR/SEM:** II/III

SUB CODE/NAME: MA8352 - LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

UNIT I – VECTOR SPACES

PART – A

1. Define vector space
2. Define subspace
3. Define span[s]
4. Define linear combinations
5. Define linearly independent and linearly dependent
6. Define basis
7. Define finite dimensional
8. State and prove cancellation law for vector addition.
9. Write down the standard basis for F^n.
10. Is $\{(1,4,-6), (1,5,8), (2,1,1), (0,1,0)\}$ is a linearly independent subset of R^3 ?
11. Write the vectors $w = (1, -2, 5)$ as a linear combination of the vectors $v_1 = (1,1,1),\ v_2 = (1,2,3)\ and\ v_3 = (2,-1,1)$
12. Determine whether $w = (4, -7, 3)$ can be written as a linear combination of $v_1 = (1,2,0)$ and $v_2 = (3,1,1)$ in R^3
13. For which value of k will the vector $u = (1, -2, k)$ in R^3 be a linear combination of the vectors $v = (3,0,-2)$ and $w = (2,-1,5)$?
14. Determine whether the set $W_1 = \{(a_1, a_2, a_3) \in R^3 : a_1 = a_3 + 2\}$ is a subspace of R^3 under the operations of addition and scalar multiplication defined on R^3
15. Point out whether the set $W_1 = \{(a_1, a_2, a_3) \in R^3 :\ a_1 - 4a_2 - a_3 = 0\}$ is a subspace of R^3 under the operations of addition and scalar multiplication defined on $R3$

16. Point out whether $w = (3,4,1)$ can be written as a linear combination of $v_1 = (1, -2,1)$ and $v_2 = (-2, -1,1)$ in

17. Show that the vectors {(1,1,0), (1,0,1) and (0,1,1)} genarate F^3

18. Determine which of the following sets are basis for R^3

$(i)\quad \{(1,0,-1),(2,5,1),(0,-4,3)\} \quad (ii)\quad \{(1,-3,-2),(-3,1,3),(-2,-10,-2)\}$

19. Determine which of the following sets are basis for $P_2(R)$

(i) $\{-1 - x + 2x^2,\ 2 + x - 2x^2,\ 1 - 2x + 4x^2\}$

(ii) $\{-1 + 2x + 4x^2,\ 3 - 4x - 210,\ -2 - 5x - 6x^2\}$

20. Evaluate which of the following sets are bases for R^3 :

(i){(1,0, −1), (2,5,1), (0, −4,3)} (ii){(−1,3,1), (2, −4, −3), (−3,8,2)}

PART – B (All are 8- marks)

I- Vector Spaces And Sub-Spaces

1. In any vector space V, the following statements are true,

(i) $0x = 0, for\ each\ x \in v$

(ii) $a0 = 0, \forall a \in v$

(iii) $(-a)x = -(ax), \forall a \in F, \forall x \in v$

(iv) if $a \neq 0, then\ ax = 0 \Longrightarrow 0$

2. Let V be the set of all polynomials of degree less than or equal to 'n' with real coefficients. Show that V is a vector space over R with respect to polynomial addition and usual multiplication of real numbers with a polynomial

(or)

Prove that for $n > 0$, the set P_n of polynomials of degree at most n consists of all polynomials of the from $a_0 + a_1x + a_2x^2 + \cdots \ldots \ldots \ldots . + a_nx^n$ is a vector space.

3. Let V denote the set of ordered pairs of real numbers. If (a_1, a_2) and (b_1, b_2) are elements of V and $c \in R, define\ (a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)\ and\ c(a_1, a_2) = (ca_1, ca_2)$. Is V a vector space over R with these operations? Justify your answer.

4. Prove that any intersection of subspaces of a vector space V is a subspaces of V.

5. The intersection of the subspaces $w_1\ and\ w_2$ of the vector space V is also a subspace.

6. Prove that the span of any subset S of a vector space V is a subspace of V. moreover, any subspace of V that contains S must also contain the span of S.

(or)

The linear span L(S) of any subset of a vector space V(F) is a subspace of V(F). moreover $L(S) \subset W$.

7. Let W_1 and W_2 be subspaces of vector space V. prove that $w_1 \cup w_2$ is a subspace of V if and only if $w_1 \subseteq w_2\ (or)\ w_2 \subseteq w_1$

(or)

Let w_1 and w_2 be sub-spaces of vector space V, prove that $w_1 \cup w_2$ is a sub-space of V, iff one is contained in the other.

8. Show that the set $w = \{(a_1, a_2, a_3) \in R^3: 2a_1 - 7a_2 + a_3 = 0\}$ is a sub-space of V

9. Show that the set $w = \{(a_1, a_2, a_3) \in R^3: a_1 + 2a_2 - 3a_3 = 0\}$ is a sub-space of V

10. Prove that $w_1 = \{(a_1 , a_2, \dots \dots \dots \dots a_n) \in F^n;\ a_1 + a_2 + \cdots \dots \dots \dots + a_n = 0\}$ is a subspace of F^n, but $w_2 = \{(a_1 , a_2, \dots \dots \dots \dots a_n) \in F^n;\ a_1 + a_2 + \cdots \dots \dots \dots + a_n = 1\}$ is not a subspace .

11. Prove that of W is a subspace of a vector space V and $w_1 , w_2, \dots \dots \dots \dots , w_n$ are in W, then $a_1w_1 + a_2w_2 + \cdots \dots \dots \dots + a_nw_n \in W$ for any scalars $a_1 , a_2, \dots \dots \dots \dots a_n$

12. Show that W is in the subspace of R^4 spanned by v_1, v_2, v_3, where $w = \begin{bmatrix} 9 \\ -4 \\ -4 \\ 7 \end{bmatrix}$,

$$v_1 = \begin{bmatrix} 8 \\ -4 \\ -3 \\ 9 \end{bmatrix}, v_2 = \begin{bmatrix} -4 \\ 3 \\ -2 \\ -8 \end{bmatrix}, v_3 = \begin{bmatrix} -7 \\ 6 \\ -18 \end{bmatrix}$$

13. If S and T are subsets of vector space V(F) ,then prove that
(i) $S \subset T \Longrightarrow L(s) \subset L(T)$ (ii) $L(S \cup T) \Longrightarrow L(S) + L(T)$ (iii) $L[L(S)] = L(S)$

II-Linear Independent, Linear Dependent and basis

1. For each of the following list of vectors in R^3.Determine whether the first vector can be expressed as a linear combination of the other two (i) (-2,0,3),(1,3,0),(2,4,-1)
(ii) (3,4,1),(1,-2,1),(-2,-1,1).

2. For each of the following list of $P_3(R)$.Determine whether the first vector can be expressed as a linear combination of the other two

(i) $x^3 - 3x + 5, x^3 + 2x^2 - x + 1, x^3 + 3x^2 - 1$

(ii) $x^3 - 8x^2 + 4x\ , x^3 - 2x^2 + 3x + 1, x^3 - 2x + 3$

(iii) $4x^3 + 2x^2 - 6\ , x^3 - 2x^2 + 4x + 1, 3x^3 - 6x^2 + x + 4$

3. Determine whether the following sets are linearly independent (or) linearly dependent.

(i) $\left\{\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ -4 & 4 \end{bmatrix}\right\} in\ M_{2x2}(R)$

(ii) $\left\{\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 2 & -2 \end{bmatrix}\right\} in\ M_{2x2}(R)$

(iii) $\left\{\begin{bmatrix} 1 & -3 & 2 \\ -4 & 0 & 5 \end{bmatrix}, \begin{bmatrix} -3 & 7 & 4 \\ 6 & -2 & -7 \end{bmatrix}, \begin{bmatrix} -2 & 3 & 11 \\ -1 & -3 & 2 \end{bmatrix}\right\}\ in\ M_{2x3}(R)$

4. Determine the following sets are linearly independent (or) linearly dependent

(i) $\{x^3 + 2x^2\ , -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\} in\ \ P_3(R)$

(ii) $\{x^3 - x\ , 2x^2 + 4\ , -2x^3 + 3x^2 + 2x + 6\} in\ \ P_3(R)$

(iii) $\{(1,-1,2), (1,-2,1), (1,1,4)\}\ in\ R^3$

(iv) $\{(1,-1,2), (2,0,1), (-1,2,-1)\}\ in\ R^3$

5. Prove that vectors $u_1 = (2,-3,1),\ u_2 = (1,4,-2),\ u_3 = (-8,12,-4),$ $u_4 = (1,37,-17), u_5 = (-3,-5,8)$ Generate R^3. Find a subset of the set $\{u_1, u_2, u_3, u_4\}$ that is a basis for R_3.

6. In each part ,determine whether the given vector is in the span S

(i) $(-1,2,1), S = \{(1,0,2), (-1,1,1)\}$

(ii) $(-1,1,1,2), S = \{(1,0,1,-1), (0,1,1,1)\}$

(iii) $-x^3 + 2x^2 + 3x + 3,\ S = \{x^3 + x^2 + x + 1, x^2 + x + 1, x + 1\}$

7. Let $S = \{(1,1,0), (1,0,1), (0,1,1)\}$ be a subset of the vector space F^3.prove that if F=R, then S is linearly independent.

8. Show that the set $\{1, x, x^2, \dots \dots \dots \dots . x^n\}$ is a linearly independent in $P_n(F)$.

9. Let V be a vector space, and let $S_1 \subseteq S_2 \subseteq V$. If S_1 is linearly dependent, then S_2 is linearly dependent.

10. Let S be a linearly independent subset of a vector space V, and V be a vector in v that is not in S. then $S \cup \{V\}$ is linearly dependent iff $V \in span\ (S)$.

11. Let V be a vector space over a field of characteristic not equal to zero, let U and V be distinct vectors in V. Prove that $\{u, v\}$ is linearly independent iff $\{u + v, u - v\}$ is linearly independent.

12. Let V be a vector space over a field of characteristic not equal to zero, let U ,V and W be distinct vectors in V. Prove that $\{u, v, w\}$ is linearly independent iff $\{u + v, v + w, u + w\}$ is linearly independent.

13. Let U,V and W be distinct vectors of a vector space V. Show that if $\{u, v, w\}$ is a basis for V, then $\{u + v + w, v + w, u\}$ is also basis for V.

14. If a vector space V is generated by a finite set S, then some subset of S is a basis for V. Hence V has a finite basis.

15. Let V be a vector space and $B = \{u_1, u_2, \dots \dots . , u_n\}$ be a subset of V, then B is a basis for V. If each $v \in V$ can be uniquely expressed as a linear combination of vector of B. (i.e) It can be expressed in the form $V = a_1u_1 + a_2u_2 + \cdots \dots . + a_nu_n$ for unique scalar $a_1, a_2, \dots \dots . , a_n$.

16. If W_1, W_2 are two subspaces of a finite dimensional vector space V then $\dim(W_1 + W_2) = \dim W_1 + dimW_2 - \dim(W_1 \cap W_2)$ and hence deduce that if $V = W_1 + W_2$, then $\dim(V) = \dim W_1 + dimW_2$.

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UNIT II – LINEAR TRANSFORMATION AND DIAGONALIZATION

PART – A

1. If $T: V \to W$ be a linear transformation then prove that $T(0) = 0'$ where 0 and $0'$are the zero elements of V and W respectively Define Subspace of a vector space
2. If $T: V \to W$ be a linear transformation then prove that $T(-v) = -v$ for $v \in V$
3. If $T: V \to W$ be a linear transformation then prove that $T(x - y) = x - y$ for all $x, y \in V$
4. Prove that the transformation T is linear if and only if $T(cx + y) = cT(x) + T(y)$
5. Illustrate that the transformation $T: R^2 \to R^2$ defined by $T(a_1, a_2) = (2a_1 + a_2, a_2)$ is linear
6. Evaluate that the transformation $T: R^3 \to R^2$ defined by by

   T $(a_1, a_2, a_3) = (a_1 - a_2, a_1 - a_3)$ is linear.
7. Describe explicitly the linear transformation $T: R^2 \to R^2$such that $T(2,3) = (4,5) and$ $T(1,0) = (0,0)$
8. Illustrate that the transformation $T: R^2 \to R^3$defined by $T(x, y) = (x + 1, 2y, x + y)$ is not linear
9. Is there a linear transformation $T: R^3 \to R^3$such that $T(1,0,3) = (1,1)$ and $(-2,0,-6) = (2,1)$?
10. Define matrix representation of T relative to the usual basis {ei}
11. Find the matrix [T]e whose linear operator $is\ T(x, y) = (5x + y, 3x - 2y)$
12. Find the matrix representation of T whose basis is $f1 = (1,2)$ $f2 = (2,3)$ such that $(x, y) = (2y, 3x - y)$
13. Define diagonalizable of a matrix with linear operator T.

14. Find the matrix representation of usual basis {ei} to the linear operator $(x, y, z) = (2y + z, x - 4y, 3x)$

15. Define eigen value and eigen vector of linear operator T.

16. State Cayley-Hamilton Theorem

17. Find the matrix A whose minimum polynomial is $t^3 - 5t^2 + 6t + 8$

18. Suppose λ is an eigen value of an invertible operator T.

Show that λ^{-1} is an eigen value of $^{-1}$.

PART – B (All are 8- marks)

1. Let V and W be the vector spaces and $T: V \rightarrow W$ be linear. Then prove that N(T) and R(T) are subspaces of V and W respectively.

2. Let V and W be vector spaces and let $T: V \rightarrow W$ be a linear transformation. If $\beta = \{v_1, v_2, \dots, v_n\}$is a basis for V,then show that $SpanT(\beta) = R(T)$. Also prove that T is one –to-one if and only if $N(T) = \{0\}$

3. Dimension theorem: Let V and W be the vector spaces and $T: V \rightarrow W$ be linear. If V is finite-dimensional then nullity(T)+rank(T)=dim(V).

4. Let V and W be the vector spaces and $T: V \rightarrow W$ be linear, then T is one-to-one iff N(T)={0}.

5. Let $T: R^3 \rightarrow R^2$ defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$. Find the basis for N(T) and the nullity of T.

6. Let $T: R^2 \rightarrow R^3$ defined by $T(a_1, a_2) = (a_1 + a_2, 0, 2a_1 - a_2)$. Find the basis for and compute N(T).

7. Let $T: R^3 \rightarrow R^2$ defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$. Find the basis forR(T) and compute R(T).

8. Let $T: R^2 \rightarrow R^3$ defined by $T(a_1, a_2) = (a_1 + a_2, 0, 2a_1 - a_2)$. Find the basis for and compute N(T).

9. Let $T: R^3 \rightarrow R^2$,defined by $T(x, y, z) = (x + y, y + z)$ then $B_1 = \{(-1,1,1), (1,-1,1), (1,1,-1)\}$ and $B_2 = \{(1,0), (0,1)\}$ then find $[T]_{B_1}^{B_2}$

10. Let $T: R^2 \rightarrow R^3$,defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$ then and $B_1 = \{(1,0), (0,1)\}$ and $B_2 = \{(1,0,0), (0,1,0), (0,0,1)\}$.

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11. Let $T: R^3 \to R^3 and\ U: R^3 \to R^3$ be the linear transformation respectively defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)\ and\ U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$. Then prove that $[T + U]_\beta^\gamma = [T]_\beta^\gamma + [U]_\beta^\gamma$

12. Let T be the linear operator on R3 defined by $T\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 4a_1 + a_3 \\ 2a_1 + 3a_2 + 2a_3 \\ a_1 + 4a_3 \end{bmatrix}$. Determine the Eigenspace of T corresponding to each Eigenvalue. Let B be the standard ordered basis for R^3.

13. Let T be a linear operator on $P_2(R)$ defined by $T[f(x)] = f(1) + f'(0)x + [f'(0) + f''(0)]x^2$. Test for diagonalisability.

14. Let T be a linear operator on a vector space V, and let $\lambda_1, \lambda_2, \lambda_3 \dots \dots \dots \lambda_k$ be distinct eigenvalues of T. For each $i = 1,2,3, \dots . k$, let Si be a finite linearly independent subset of the Eigen space $E_{\lambda i}$. Then $S = S_1 \cup S_2 \cup \dots \dots \cup S_k$ is a linearly independent subset of V.

15. Let $T: P_2(R) \to P_3(R)$ be defined by $T[f(x)] = xf(x) + f'(x)$ is linear. Find the bases for both (T), (T), nullity of T, rank of T and determine whether T is one –to-one or onto.

16. Let $T: R^3 \to R^3$ be a linear transformation defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Evaluate a basis and dimension of null space N(T) and range space R(T) and range space R(T). Also verify dimension theorem.

17. Let V and W be vector spaces over F, and suppose that $\{v_1, v_2 \dots \dots \dots . v_n\}$ is a basis for V, For $w_1, w_2 . w_n$ in W Prove that there exists exactly one linear transformation $T: V \to W$ such that $T(v_i) = w_i$ for i=1,2,…n

18. Suppose that T is one –to-one and that s is a subset of V .Prove that S is linearly independent if and only if T(S) is linearly independent .
Suppose $\beta = \{v1, v2., \dots \dots . vn\}$ is abasis for V and T is one –to-one and onto
Prove that $T(\beta) = \{T(v_1), T(v_2), T(v_n)\}$ is a basis for W

19. Let V and W be vector spaces with subspaces V_1 W_1 respectively. If $T: V \to W$ is linear .Prove that $T(V_1)$ is a subspace of w and that $x \in V: T(x) \in W_1\}$ is a subspace of V

20. If $T: R^4 \to R^3$ is a linear transformation defined $T\{x_1, x_2, , x_3, x_4\} = (x_1 - x_2 + x_3 + x_4, {}_1 + 2x_3 - x_4, {}_1 + x_2 - 3x_3 - 3x_4)$ for $\{x_1, x_2, , x_3, x_4\} \in R$ then verify Rank(T)+Nullity(T)=dim R^4 find yhe bases of N(T) and R(T)

21. For a linear operator $T: R^3 \to R^3$ defined as $T(a, b, c) = (-7a - 4b + 10c, 4a - 3b + 8c, -2a + b - 2c)$,Point out the eigen values of T and an ordered basis β for R^3 such that the matrix of the given transformation with the respect to the new resultant basis β is a diagonal matrix

22. Let T be a linear operator $(a, b, c) = (-4a + 3b - 6c, 6a - 7b + 12c, 6a - 6b + 11c)$, be the ordered basis then find $[T]$ which is a diagonal matrix.

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UNIT III – INNER PRODUCT SPACES

PART – A

1. Define inner Product Space and give its axioms.
2. Define norm
3. Define orthogonal
4. Define orthonormal
5. Define orthogonal complement.
6. Define adjoint operator.
7. Let $x = (2,1+i,i) and\ y = (2-i,2,1+2i)$ be a vector in C^3 compute $< x,y >$.
8. Find the norm and distance between the vectors u=(1,0,1) and v=(-1,1,0)
9. Find the norm of the vector u=(1,-1,1) and v=(-1,1,0) in R^3 with respect to the inner product defined by $< u,v > = u_1v_1 + 2u_2v_2 + 3u_3v_3$.
10. Find the norm of v = (3,4) ∈ R^2 with respect to the usual product.
11. Consider $f(t0 = 3t - 5\ and\ g(t) = t^2$ in the polynomial space p(t) with inner product $< f,g > = \int_0^1 f(t0g(t)dt, then\ find\ \|f\|\ and\ \|g\|$.
12. Prove that in an inner product space V, for any $u,v \in V.\ \|u+v\|^2 + \|u+v\|^2 = 2\|u\|^2 + 2\|u\|^2$
13. In ([0,1]), let $f(t) = t$, $g(t) = e^t$ Evaluate $< f, g >$.
14. Let R2 and $S = \{(1,0),(0,1)\}$. Check whether S is orthonormal basis or not.
15. Let $S = \left\{\left(\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}}\right),\left(\frac{2}{\sqrt{5}},\frac{-1}{\sqrt{5}}\right)\right\}$ .verify S is orthonormal basis or not.
16. Find the value of 'a' if the vectors (2,a) and (6,4) are orthogonal vectors in R^2.
17. Find 'k' so that $u = (1,2,k,3)\ and\ v = (3,k,7,-5)$ in R^4 are orthogonal.
18. If $u = (2,1,2)\ and\ v = (1,2,1)\ find\ proj(v,u)$.

19. State Cauchy Schwarz inequality and Triangle inequality.

20. If x, y and z are vector of inner product space such that $< x, y > =< x, z >$then prove that $y = z$.

21. Prove that the norm in a inner product space satisfies $\|v\| \geq 0 \; and \; \|v\| = 0 \; if \; and \; only \; if \; v = 0.$

22. Find the norm of $v = (1,2) \in R^2$with respect to the inner product $< u, v > = x_1y_1 - 2x_1y_2 - 2x_2y_1$

23. Let S={(1,0, i)(1,2,1)} in c^3 Point out $S^{\perp}$

24. Let W= span ({i,0,1}) in c^3 find the orthonormal bases of w and $w^{\perp}$

25. Let w be a subspace of v then prove that v=w ⊕ w.

26. Let T be a linear operator on v,β is an orthonormal basis then prove that $[T^*]_\beta$=$[T]_\beta$

27. Let S and T be linear operators on V then prove that $(S + T)^* = S^* + T^*$

28. Show that $I^* = I \; for \; every \; u, v \in v$

29. Let T be a linear operator on v and let W be a T invariant subspace of V.

Show that w is invariant under T^*

30. Let $V = R2, T(a, b) = (2a + b, a - 3b)$ $x = (3,5)$ find T^*at the given vector in V, when T is a Linear operator.

PART – B (All are 8- marks)

1. let $V = M_{mxn}(F) and \; define < A, B >= tr \; (B^* A)$ for $A, B \in V$,the trace of a matrix A is defined by $tr(A) = \sum_{i=1}^{k} A_{ii}$. Verify <. , .> is an inner product space.

2. Let V be a real (or) complex vector space and let B be a basis for V for $x, y \in V$ there exists $v_1, v_2, \dots \dots \dots . . v_n \in B \; such \; that \; x = \sum_{i=1}^{n} a_i v_i \; and \; y = \sum_{i=1}^{n} b_i v_i$.
Define $< x, y > = \sum_{i=1}^{n} a_i \overline{b_i}$ .prove that <. , .> is an inner product on V and that B is an orthonormal basis V.

3. Let $x = (2,1 + i, i \,) and \; (2 - i, 2,1 + 2i)$ be vectors in C^3.compute (i) $< x, y >$ (ii) $\|x\|$ (iii) $\|y\|$ (iv) $\|x + y\|$ (v) Cauchy's inequality (vi) Triangle inequatily.

4. Let V be an inner product space. Then for $x, y, z \in V \; andc \in F$,the following statements are true.

i) $< x, y + z >= < x, y > +< x, z >$
ii) $< x, cy >= \bar{c} < x, y >$
iii) $< x, 0 >=< 0, x >= 0$
iv) $< x, x >=< x, z > \; \forall x \in v, then \; y = z$

5. Let V be an inner product space over F, then for all $, y \in V \; and \; c \in F$, the following statements are true

i) $\|cx\| = |c|\|x\|$
ii) $\|x\| = 0, iff \; x = 0$
iii) $|< x, y >| \leq \|x\|\|y\|$
iv) $\|x = y\| \leq \|x\| + \|y\|$

6. Let T be a linear operator on an inner product space V and suppose that $\|T(x)\| = \|x\|$ for all X , prove that T is one-to-one.

7. State and prove Cauchy-Schwarz inequality and Triangle inequality in an inner product space.

8. Let V be an inner product space. Prove that
(a) $\|x \pm y\|2 = \|x\|2 \pm 2R < x, y > +\|y\|^2$ for all x, y ∈ V, where $R < x, y >$ denotes the real part of the complex number $< x, y >$.
(b) $|\|x\| - \|y\||^2 \leq \|x - y\|$ for all x, y ∈ V.

9. Let V be an inner product space over F. prove that polar identities for all $x, y \in V$.
$< x, y > = \frac{1}{4}\|x + y\|^2 - \frac{1}{4}\|x - y\|^2$ if F=R.

10. show that in R^3, the vectors (1,1,0), (1,-1,1), (-1,1,2) are orthogonal, are they orthonormal? Justify.

11. Let V be the vector space of polynomial with inner product given by
$< x, y > = \int_0^1 f(t)g(t)dt$. let $(t) = t + 2 \; and \; g(t) = t^2 - 2t - 3$,
find (i) $< f, g >$ (ii) $\|f\|$ (iii) $\|g\|$.

12. In $([0,1]), let \; f(t) = t \; and \; g(t) = et. Compute < f, g >, \|f\|, \|g\| and \|f + g\|$
Then verify both the Cauchy-Schwarz inequality and the triangle inequality

13. Let $\{v_1, v_2, \dots \dots \dots . . v_k\}$ be an orthogonal set in V and $a_1, a_2, \dots \dots \dots . . a_k$ be scalars,
Prove that $\left\|\sum_{i=1}^{k} a_i v_i\right\|^2 = \sum_{i=1}^{k} |a_i|^2 \|v_i\|^2$.

14. Apply the Gram-Schmidt process to the given subsets $S = \{(1,0,1), (0,1,1), (1,3,3)\} \; and \; x = (1,1,2)$ of the inner product space $V = R^3$

i) To obtain an orthogonal basis for span (s)
ii) Normalize the vectors in the basis to obtain an orthonormal basis for span (s)
iii) Compute to Fourier coefficients of the given vector.

15. Let $V = R^4$, $let \; w_1 = (1,0,1,0)$, $w_2 = (1,1,1,1), w_3 = (0,1,2,1)$. Use Gram-schmidt process to compute the orthogonal vectors and normalize these vectors.

16. Evaluate using the Gram Schmidt Process to the given subset
$S = \{(1,1,1),(0,1,1),(0,0,1)\}$ $and\ x = (1,0,1)$ of the inner product space $V = R^3$ to obtain (i) an orthogonal basis for span(S).
(ii) Then normalize the vectors in this basis to obtain an orthonormal basis β for span(S)
(iii) compute the Fourier coefficients of the given vector relative to β.

17. Evaluate by the Gram Schmidt Process to the given subset
$S = \{(1,-2,-1,3),(3,6,3,-1),(1,4,2,8)\}$ $and\ x = (-1,2,1,1)$ of the inner product space $= R^4$ to obtain (i) an orthogonal basis for span(S).
(ii)Then normalize the vectors in this basis to obtain an orthonormal basis β for span(S)
(iii)compute the Fourier coefficients of the given vector relative to β.

18. Apply the Gram-Schmidt process to the given subsets S of the inner product space V to obtain

i) orthogonal basis for span (s)
ii) Normalize the vectors in the basis to obtain an orthonormal basis for span(s)

Let $V = P_2(R)$ with the inner product $< f,g > = \int_0^1 f(t)g(t)dt,$ and $\{1,x,x^2\}$

19. Apply the Gram-Schmidt process to the given subsets
$S = \{(1,i,0),(1-i,2,4i)\}$ $and\ x = (3+i,4i,-4)$ of the inner product space V= R^3

i) To obtain an orthogonal basis for span (s)
ii) Normalize the vectors in the basis to obtain an orthonormal basis for span (s)
iii) Compute to Fourier coefficients of the given vector.

20. State and prove the Gram-Schmidt orthogonalization theorem (or)
Let V be an inner product space and $S = \{w_1, w_2, \ldots \ldots . w_n\}$ be a linearly independent subset of V. Define $S' = \{v_1, v_2, \ldots \ldots . v_n\}$ when $v_1 = w_1$ and $v_k = w_k - \sum_{j=1}^{k-1} \frac{<w_k, v_j>}{\|vj\|^2} vj$ for $2 \le k \le n$. Then S' is an orthogonal set of non-zero vectors such that Span (s') = Span (s).

21. Let V be an inner product space, $S\ and\ S_0$ be the subset of V and W be a finite dimensional subspace of V. Prove that the following results

i) $S_0 \subseteq S \implies S^{\perp} \subseteq {S_0}^{\perp}$
ii) $S \subseteq (S^{\perp})^{\perp}$ $, so\ span\ (s) \subseteq (S^{\perp})^{\perp}$
iii) $W = (W^{\perp})^{\perp}$
iv) $V = W \oplus W^{\perp}$

22. Let V be a finite dimensional inner product space, and let T be a linear operator on V. Then there exists a unique function $T^*: V \to V$ such that
$< T(x), y > = < x, T^*(y) >$ $for\ all\ x, y \in V$ and T^* is linear.

23. Let V be an inner product space, T and U be linear operators on V.then

i) $(T+U)^* = T^* + U^*$

ii) $(CU)^* = \bar{C}T^*, for\ any\ c \in F$

iii) $(TU)^* = U^*T^*$

iv) $T^{**} = T$

v) $I^* = I$

24. $Let\ A\ and\ B\ be\ n\ X\ n\ matrices. Then\ prove\ that$

(i) $(A + B)* = A* + B*$

(ii) $(cA)* = c\bar{A}*\ for\ all\ c \in F$

(iii) $(AB)* = B*A*$

(iv) $A** = A$

(v) $I* = I$

25. Let V be a finite dimensional inner product space and let T be a linear operator on V. Prove that if T is invertible, then T^* is invertible and $(T^*)^{-1} = (T^{-1})^*$.

26. Suppose that S=$\{v_1, v_2, \dots v_k\}$ is an orthonormal set in an n-dimensional inner product space V. Then Prove that

(i) S can be extended to an orthonormal basis $\{v_1, v_2, \dots v_k, v_{K+1} \dots \dots\ v_n\}$ for V

(ii) If W= span(S),then s_1=$\{v_{K+1}, v_{K+2} \dots \dots . v_n\}$ is an orthonormal basis for $w^{\perp}$

(iii) If W is any subspace of V , then dim(v)=dim(w)+dim($w^{\perp}$).

27. Find the least squares lines and error for the following data (1,2),(2,3),(3,5),(4,7).

28. For each of the sets of data that follows, use the least squares approximation to find the best fits with both (i) a linear function and (ii) a quadratic function. Compute the error E in both cases. {(-3, 9), (-2, 6), (0, 2),(1, 1)}

29. For each of the sets of data that follows, use the least squares approximation to find the best fits with both (i) a linear function and (ii) a quadratic function. Compute the error E in both cases.{(-2, 4), (-1, 3), (0, 1), (1, -1), (2, -3)}

30. Consider the system $x + 2y + z = 4$; $x - y + 2z = -11$; $x + 5y = 19$; find the minimal solution.

ST.ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY

(An ISO 9001:2015 Certified Institution)

Anguchettypalayam, Panruti – 607106.

QUESTION BANK

PERIOD: JULY - NOV 2018 **BATCH**: 2015 – 2019

BRANCH: ECE **YEAR/SEM:** II/III

SUB CODE/NAME: MA8352 - LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

UNIT IV - PARTIAL DIFFERENTIAL EQUATIONS

PART – A

1. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from
$log(az-1) = x + ay + b$ ii) $Z = ax + by + ab$ **(A/M-15)**

2. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from
$Z = ax + by + ab$

3. Construct the partial differential equation of all spheres whose centers lie on the Z-axis by the elimination of arbitrary constants. **(N/D-15)**

4. Find the PDE of all spheres whose radii are same. **(N/D-16)**

5. Form the partial differential equation by eliminating the arbitrary functions from
$\boldsymbol{f(x^2+y^2, z-xy) = 0}$. **(M/J-16)**

6. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b'from
$\boldsymbol{z = ax^2 + by^2}$ **(N/D-13)(A/M-17)**

7. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b'from
$z = (\boldsymbol{x^2+a^2})(\boldsymbol{x^2+b^2})$ **(A/M-16)**

8. Form the PDE by eliminating the arbitrary functions from $\boldsymbol{z = f(y/x)}$ **(N/D-14)**

9. Form the PDE by eliminating the arbitrary functions from $z = f(x^2+y^2)$

10. Form the partial differential equation by eliminating the arbitrary functions from
$i)\ \emptyset(\boldsymbol{x^2+y^2+z^2, xyz}) = \boldsymbol{0}$. ii) $i)\ \emptyset(\boldsymbol{x^2+y^2, z-xy}) = \boldsymbol{0}$

11. Form the PDE by eliminating the arbitrary functions from $\boldsymbol{z = xf(2x+y) + g(2x+y)}$.

12. Solve $\sqrt{p} + \sqrt{q} = 1$

13. Find the complete solution of $\boldsymbol{q = 2px}$ **(A/M-15)**

14. Find the complete solution of $\boldsymbol{p + q = 1}$. **(N/D-14)**

15. Find the complete integral of $\boldsymbol{\frac{z}{pq} = \frac{x}{p} + \frac{y}{q} + \sqrt{pq}}$. **(N/D-16)**

16. Find the complete solution of PDE $\boldsymbol{p^3 - q^3 = 0}$. **(M/J-16)**

17. Find the complete integral of $\boldsymbol{pq = xy}$.

18. Solve $\frac{\partial^2 z}{\partial x \partial y} = 0$.

19. Solve $\frac{\partial^2 z}{\partial x^2} = \sin x$ **(A/M-17)**

20. Solve $(D + D' - 1)(D - 2D' + 3)z = 0$. **(N/D-15)**

21. Solve $(D^3 - D^2D' - 8DD'^2 + 12D'^3)z = 0$

22. Solve $(D - D' - 1)(D - 2D' - 2)z = 0$

23. Solve $z = 1 + p^2 + q^2$.

24. Solve $\left(D^2 + 2DD' + D'^2 - 2D - 2D'\right)z = \sin(x + 2y)$.

25. Solve $\left(D^4 - D'^4\right)z = 0$.

PART – B

[First Half] (All are 8-marks)

I - Lagrange's method

1. Solve:- $(mz - ny)p + (nx - lz)q = (ly - mx)$
2. **Solve:-** $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$
3. Solve:- $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ **(A/M-17)**
4. Solve:- $(x - 2z)p + (2z - y)q = y - x$ **(N/D-17)**
5. **Solve:-** $x(y^2 + z^2)p + y(z^2 + x^2)q = z(y^2 - x^2)$ **(N/D-16)**
6. **Solve:-** $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$ **(N/D-16)**
7. Solve:- $x(y - z)p + y(z - x)q = z(x - y)$ **(A/M-18) (N/D-14)**
8. **Solve:-** $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$ **(A/M-15) (M/J-16)**
9. **Solve:-** $(z^2 - y^2 - 2yz)p + (xy + zx)q = xy - zx$ **(N/D-15) (A/M-17)**
10. **Solve:-** $(y - xz)p + (yz - x)q = (x + y)(x - y)$
11. **Solve:-** $\boldsymbol{(y^2 + z^2)p - xy\,q + xy = 0}$
12. **Solve:-** $z(x + y)p + z(x - y)q = (x^2 + y^2)$

II- solve the partial diff-equations

1. Find the S.I $Z = px + qy + \sqrt{1 + p^2 + q^2}$ **(M/J-16)**
2. Find the S.I $Z = px + qy + p^2 - q^2$ **(N/D-15)**
3. Find the S.I $Z = px + qy + p^2q^2$ **(N/D-14)**
4. Find the C.I $z^2(p^2 + q^2) = (x^2 + y^2)$ **(M/J-16)**
5. Find the C.I $p^2 + x^2y^2q^2 = x^2z^2$ **(N/D-15)**
6. Find the general solution $Z = px + qy + p^2 + pq + q^2$ **(A/M-18) (A/M-17)**
7. Find the C.I $p^2 + q^2 - 4pq = 0$
8. **Solve :-** $Z = px + qy + pq$
9. Solve :- $\sqrt{p} + \sqrt{q} = 1$
10. Solve :- $p^2 + x^2y^2q^2 = x^2z^2$ **(N/D-15)**
11. Solve :- $x^2p^2 + y^2q^2 = z^2$ **(M/J-16)**
12. Solve :- $p(1 + q) = qz$ **(M/J-16)**
13. Solve :- $p(1 + q^2) = q(z - a)$

[Second Half] (All are 8-marks)

I – Solve the homogeneous and non- homogeneous equations

1. Solve:- $\left(D^3 - 7DD^{'2} - 6D^{'3}\right)z = \sin(x + 2y)$
2. Solve:- $\left(D^2 + 4DD^{'} - 6D^{'2}\right)z = sin(x - 2y) + e^{2x-y}$ **(A/M-18)**
3. Solve:- $\left(D^2 + DD^{'} - 6D^{'2}\right)z = x^2y + e^{3x+y}$ **(N/D-14)**
4. Solve:- $\left(D^3 - 7DD^{'2} - 6D^{'3}\right)z = x^2y + \sin(x + 2y)$ **(M/J-16)**
5. Solve:- $\left(D^2 + DD^{'} - 6D^{'2}\right)z = e^{x-y} + \cos(2x + y)$ **(A/M-17)**
6. Solve:- $\left(D^3 - 7DD^{'2} - 6D^{'3}\right)z = e^{2x+y} + \sin(x + 2y)$ **(N/D-15)**
7. Solve:- $\left(D^3 - 7DD^{'2} - 6D^{'3}\right)z = e^{x+y} + \sin(x + 2y)$
8. Solve:- $\left(D^2 - DD^{'} - 2D^{'2}\right)z = (2x + 3y) + e^{3x+4y}$

9. Solve:- $\left(D^2 + 2DD' + D'^2\right)z = e^{x-y} + xy$ **(N/D-17)**

10. Solve:- $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x \partial y^2} = \cos(x - 2y) + 3xy^2$

11. Solve:- $\left(D^2 + DD' - 6D'^2\right)z = y\cos x$ **(A/M-18) (N/D-16)**

12. Solve:- $\left(D^2 + 2DD' + D'^2\right)z = 2\cos y - x\sin y$ **(N/D-15)**

13. Solve:- $\left(D^2 + D'^2\right)z = x^2y^2$ **(N/D-15)**

14. Solve:- $\left(D^2 + 2DD' + D'^2\right)z = x^2y + e^{x-y}$ **(A/M-17)**

15. Solve: $(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y.$ **(M/J-16)**

16. Solve:- $\left(D^2 - 3DD' + 2D'^2 + 2D - 2D'\right)z = \sin(2x + y)$ **(A/M-17)**

17. Solve:- $\left(D^2 + 2DD' + D'^2\right)z = x^2y$

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QUESTION BANK

PERIOD: JULY - NOV 2018 **BATCH**: 2015 – 2019

BRANCH: ECE **YEAR/SEM:** II/III

SUB CODE/NAME: MA8352 LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

UNIT V – FOURIER SERIES SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

PART – A

19. Write down Dirichlet's conditions of Fourier series. **(M/J-16)**

20. Expand $f(x) = 1, in\ (0,\pi)$ as a half range sine series. **(N/D-15)**

21. If the fourier series of the function f(x) = x, in $(-\pi < x < \pi)$ with period 2π is given by **(M/J-16)**

$f(x) = 2(\sin x - \frac{sin2x}{2} + \frac{sin3x}{3} - \frac{sin4x}{4} + \cdots)$ then find the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ ...

22. Find the sin series function $f(x) = 1, in\ 0 \le x \le \pi$ **(A/M-17)**

23. Find the value of the fourier series of $f(x) = \begin{cases} 0\ in\ (-c,0) \\ 1\ in\ (0,c) \end{cases}$ at the point of discontinuity $x = 0$. **(N/D-15)**

24. Find the value of b_n in the Fourier series of $f(x) = \begin{cases} x+\pi\ in\ (-\pi,0) \\ -x+\pi\ in\ (0,\pi) \end{cases}$. **(N/D-14)**

25. State the sufficient condition for existence of Fourier series. **(A/M-17)**

26. If $(\pi - x)^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}\ in\ 0 < x < 2\pi$,then deduce that value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ **(N/D-14)**

27. If the Fourier series of the function $f(x) = x + x^2$, in the interval $(0,\pi)$ is **(A/M-14)**

$\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{4}{n^2}\cos nx - \frac{2}{n}\sin nx\right]$, then find the value of the infinite series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$. ....

28. Find the root mean square value of $f(x) = x(l - x)\ in\ 0 \le x \le l$. **(N/D-14)**

29. Definition of root mean square value (RMS value) of a function $f(x)\ in\ a < x < b$.

30. The cosine series for $f(x) = x\,sinx$ for $0 < x < \pi$ is given as $x\,sinx = 1 - \frac{1}{2}\,cosx -$

$2\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2-1}\,cosnx$. Deduce that $1 + 2\left[\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots\right] = \frac{\pi}{2}$. (A/**M-14)**

31. State Parseval's theorem on Fourier series.

32. Find the value of a_0 in the Fourier series of $f(x) = e^x\ in\ (0,2\pi)$

33. If $f(x) = x^2 \; in \; (-l, l)$, find the value of a_0 in the Fourier series.

34. Expand $f(x) = k, in \; (0, \pi)$ as a half range sine series.

35. State the assumptions in deriving one-dimensional wave equation. **(N/D-16)**

36. State the three possible solution of the one-dimensional wave equation. **(A/M-14)**

37. State the three possible solution of the one-dimensional heat equation $u_t = \alpha^2 u_{xx}$. **(N/D-16)**

38. Write down the various possible solution of the one-dimensional heat equation. **(M/J-16)**

39. Classify the equation $u_{xx} + u_{xy} = f(x, y)$. **(M/J-16)**

40. Write all possible solution of the two-dimensional heat equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. **(N/D-15)**

41. Solve $3x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$ using method of separation of variables. **(N/D-15)**

42. What is the constant α^2 in the wave equation.

43. In the wave equation $\frac{\partial^2 x}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ what does c^2 stand for ?

44. What is mean by steady state condition in heat flow?

45. In steady state conditions derive the solution of one dimensional heat flow?

46. Difference between one dimensional wave and heat flow equations?

47. The PDE of one dimensional heat equation is $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$. what is α^2?

48. What are the assumptions made in deriving one-dimensional heat equation?

49. State one-dimensional heat equation with the initial and boundary conditions?

50. State one-dimensional wave equation (zero initial velocity) with the initial and boundary conditions?

PART – B

[First Half] (All are 8- marks)

I- Find the ODD and **EVEN** type in the interval $[-\boldsymbol{\pi}, \boldsymbol{\pi}]$ **and** $[-\boldsymbol{l}, \boldsymbol{l}]$

1. **Find the Fourier series** $\boldsymbol{f(x) = x^2}$ $, in -\pi < x < \pi$ **. Hence deduce the value of** $\sum_{n=1}^{\infty} \frac{1}{n^2}$.
2. $\boldsymbol{f(x) = x^2}$ $, in \; the \; interval \; [-\pi, \pi]$ and deduce that

   (i) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \ldots \ldots = \frac{\pi^2}{6}$ (ii) $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \ldots .. = \frac{\pi^2}{12}$

   (iii) $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots = \frac{\pi^2}{8}$ **(N/D-14)**

3. Expand $f(x) = x^2$ as a Fourier series in the interval ($-\boldsymbol{\pi}, \boldsymbol{\pi}$) and

   Hence deduce that $\frac{\mathbf{1}}{\mathbf{1^4}} + \frac{\mathbf{1}}{\mathbf{2^4}} + \frac{\mathbf{1}}{\mathbf{3^4}} + \frac{\mathbf{1}}{\mathbf{4^4}} + \cdots \ldots \ldots \ldots .. = \frac{\boldsymbol{\pi^4}}{\mathbf{90}}$ **(N/D-16)**

4. Find the Fourier series of $\boldsymbol{f(x) = x \;\; in - \pi < x < \pi}$ of periodicity of 2π **(M/J-16)**
5. Find the Fourier series of $\boldsymbol{f(x) = |x| \;\; in - \pi < x < \pi}$ of periodicity of 2π **(M/J-16)**
6. Find the Fourier series of $\boldsymbol{f(x) = x} + x^2 \;\; \boldsymbol{in - \pi < x < \pi}$

   and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ **(N/D-14 , A/M-17)**
7. Find the Fourier series of $\boldsymbol{f(x) = |\sin x| \; in - \pi < x < \pi}$ of periodicity of 2π **(A/M-15)**
8. Find the Fourier series of $\boldsymbol{f(x) = |\cos x| \; in - \pi < x < \pi}$ **(M/J-16)**
9. Find the Fourier series expansion the following periodic function

   $$f(x) = \begin{cases} l + x \;, & -l < x < 0 \\ l - x \;, & 0 < x < l \end{cases}$$

10. Find the Fourier series expansion the following periodic function

    $f(x) = \begin{cases} 2 + x \;, & -2 < x < 0 \\ 2 - x \;, & 0 < x < 2 \end{cases}$ .Hence deduce that $\mathbf{1} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots = \frac{\pi^2}{8}$ **(N/D-15)**

II-Half-Range Series

(a) Find the Cosine Series

1. Find the half range cosine series of $\mathbf{f(x) = (\pi - x)^2, 0} < x < \boldsymbol{\pi}$.

   Hence find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots \ldots \ldots \ldots$ **(N/D-15)**
2. Find the half range cosine series of $\mathbf{f(x) = x \;\; in \; (0, \pi)}$.

   Hence deduce that the value $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots$ **(N/D-17)**
3. Find the half range cosine series of $\mathbf{f(x) = (x - 1)^2, \; in \; 0 < x <} \mathit{1}$. **(N/D-14)**
4. Obtain the Fourier cosine series expansion of $\mathbf{f(x) = x(\pi - x)}$, $\mathbf{in \; 0 < x <} \pi$. **(A/M-14)**
5. Expand $\mathbf{f(x)} = \begin{cases} \mathbf{x} \;, & \mathbf{0} < \mathbf{x} < 1 \\ \mathbf{2 - x} \;, & \mathbf{1} < \mathbf{x} < 2 \end{cases}$ as a series of cosine in the interval **(0,2)** **(N/D-16) (A/M-17)**

6. Find the half range cosine series of $\mathbf{f(x) = x \sin x}$, in the interval $[\mathbf{0}, \boldsymbol{\pi}]$ **(N/D-11)**

(b) Find the Sine series

1. Find the half- range sine series of $\mathbf{f(x)} = \begin{cases} \mathbf{x}, & \mathbf{0} \leq \mathbf{x} \leq \frac{\pi}{2} \\ \boldsymbol{\pi} - \mathbf{x}, & \frac{\pi}{2} \leq \mathbf{x} \leq \boldsymbol{\pi} \end{cases}$.

   Hence deduce the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ **(A/M-15)**
2. Find the half- range sine series of $\mathbf{f(x)} = \begin{cases} \mathbf{x}, & \mathbf{0} \leq \mathbf{x} \leq \mathbf{1} \\ \mathbf{2 - x}, & \mathbf{1} \leq \mathbf{x} \leq \mathbf{2} \end{cases}$

3. Find the half-range sine series of $\mathbf{f(x) = x \sin x}$, in the $[\mathbf{0}, \boldsymbol{\pi}]$ **(M/J-16)**

IV- Find the Fourier series in the interval (0,2l)

1. Find the Fourier series of period $2l$ for the function $f(x) = (l - x)^2, \mathbf{0 < x < 2}l$

   $Deduce\ the\ sum\ \sum_{n=1}^{\infty} \frac{1}{n^2}$

2. Find the Fourier series of period $2l$ for the function $f(x) = \begin{cases} l - x, 0 < x < l \\ 0 \ \ , l < x < 2l \end{cases}$

V-Find the Fourier series in the interval (0,2π)

1. **Find the Fourier series for the function** $\boldsymbol{f(x) = \frac{1}{2}(\pi - x)},\ \boldsymbol{in\ interval\ 0} < \boldsymbol{x} < 2\boldsymbol{\pi}.$

   $Deduce\ the\ sum\ 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \ldots \ldots \ldots$

2. **Find the Fourier series for the function** $\boldsymbol{f(x) = (\pi - x)^2},\ \boldsymbol{in\ interval\ 0} < \boldsymbol{x} < 2\boldsymbol{\pi}.$

   $Deduce\ the\ sum\ 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \ldots \ldots \ldots$

3. Find the Fourier series of period 2π for the function $\boldsymbol{f(x) = x\cos x}$ **in** $\mathbf{0} < x < 2\pi$ **(A/M-17)**
4. Find the Fourier series of period 2π for the function $f(x) = \begin{cases} 1, 0 < x < \pi \\ 2 \ \ , \pi < x < 2\pi \end{cases}$ **(N/D-11)**

[Second Half] (All are 16-marks)

TYPE-I (String with zero-velocity)

1. A string is stretched and fastened at two points $x = 0\ and\ x = l$ motion is started by displacing the string into the form $\boldsymbol{y = k(lx - x^2)}$ from which is $t = 0$. Find the displacement of the time 't'.
   (A/M-15)
2. A tightly stretched string of length $2l$ is fastened at both ends, the midpoint of the string is displaced by a distance $'h'$ transversely and the string is released from rest in this position. Find the displacement of the string at any time 't'. **(A/M-17)**
3. A tightly stretched string with fixed end points $x = 0\ and\ x = l$ is initially in a position given by $\boldsymbol{y(x, 0) = y_0\ \ sin^3\left(\frac{\pi x}{l}\right)}$,If it is released from rest from this position. Find the displacement of the end time 't'.

4. A tightly stretched flexible string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$, The string is given a shape defined by $\boldsymbol{f(x) = kx^2(l - x)}$, where 'k' is a constant, and then released from rest. Find the displacement of any point *'x'* of the string at any time $t > 0$.

5. Find the displacement of any point of a string, if it is of length $2l$ and vibrating between fixed end points with initial velocity zero $\boldsymbol{f(x)} = \begin{cases} \frac{kx}{l} & , \ in\ 0 < x < l \\ 2k - \frac{kx}{l} & , in\ l < x < 2l \end{cases}$

6. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $\boldsymbol{y(x, 0) = k \sin\frac{3\pi x}{l}\cos\frac{2\pi x}{l}}$. If it is released from rest from this position, determine the displacement $y(x, t)$.

TYPE-II (String with non-zero velocity)

1. If a string of length *'l'* is initially at rest in its equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = \boldsymbol{V_0 sin^3\left(\frac{\pi x}{l}\right)}, (0 < x < l)$. Determine the displacement function $y(x, t)$ at any time *'t'*. **(N/D-14)**

2. Find the displacement of a string stretched between two fixed points at a distance of $2l$ apart when the string is initially at rest in equilibrium position and points of the string are given initial velocity V, where $\boldsymbol{V = f(x)} = \begin{cases} \frac{x}{l} & , \ in\ 0 < x < l \\ \frac{(2l - x)}{l} & , in\ l < x < 2l \end{cases}$, x being the distance from an end point.

(A/M-16)

3. If a string of length *'l'* is initially at rest in its equilibrium position and each of its points is given the velocity V. Such that $= \begin{cases} \frac{2kx}{l} & , 0 < x < \frac{l}{2} \\ \frac{2k(l - x)}{l} & , \frac{l}{2} < x < l \end{cases}$. Find the displacement function $y(x, t)$ at any time *'t'*.

4. A tightly stretched string of length 'l' with fixed end points is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $\boldsymbol{y_t(x, 0) = v_0 \sin\left(\frac{3\pi x}{l}\right)\cos\left(\frac{\pi x}{l}\right)}$. Where $0 < x < l$. Find the displacement of the string at a point, at a distance x from one at any instant 't' **(N/D-16)**

5. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $V = \lambda x(l - x)$ then, Show that $\boldsymbol{y(x,t)} = \frac{8\lambda l^3}{\pi^4}\sum_{n=1,3,5}^{\infty}\frac{1}{n^4}\boldsymbol{sin}\left(\frac{n\pi x}{l}\right)\mathbf{sin}\left(\frac{n\pi at}{l}\right)$.

6. If a string of length *'l'* is initially at rest in its equilibrium position and each of its points is given the velocity V. Such that $= \begin{cases} \boldsymbol{kx} & ,\mathbf{0} < x < \frac{l}{2} \\ \boldsymbol{k(l-x)}, & \frac{l}{2} < x < l \end{cases}$. Find the displacement function $y(x,t)$ at any time *'t'*. **(N/D-15)**

7. A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given initial velocities V, where $\boldsymbol{V = f(x)} = \begin{cases} \frac{cx}{l} & , \ \boldsymbol{in\ 0} < x < l \\ \frac{c}{l}(\boldsymbol{2l - x}) & , \boldsymbol{in\ l} < x < 2\boldsymbol{l} \end{cases}$, x being the distance from an end point. Find the displacement of the string at any time.

ONE DIMENSIONAL HEAT EQUATIONS

1. A bar 10 cm long with insulated sides has its ends A and B maintained at temperature at 50^0c and 100^0c, respectively, until steady state conditions prevails. The temperature at A is suddenly raised to 90^0c and at the same time lowered to 60^0c at B. Find the temperature distributed in the bar at time 't.' **(N/D-15)**

2. A long rectangular plate with insulated surface is l cm wide. If the temperature along one short edge is $(\boldsymbol{x},\ \mathbf{0})$ = $(\boldsymbol{lx} - \boldsymbol{x^2})$ $\mathbf{0} < x < l$, while the other two long edges = 0 and $x = l$ as well as the other short edge are kept at 0^0c. Find the steady state temperature function $u(x, y)$

3. A rod 30cm long has its ends A and B kept at 20^0c and 80^0c respectively until steady state conditions previl the temperature at each end its them suddenly reduced at 0^0c and kept so. Find the resulting temperature function (x, t) taking x=0 at A.

4. The ends A and B of a rod 30cm long have their temperature at 20^0c and 80^0c until steady state conditions prevail. The temperature of the end B is suddenly reduced to 60^0c and kept so while the end A is raised to 40^0c. Find the temperature distribution in the rod after time.

TWO DIMENSIONAL HEAT EQUATIONS

1. A rectangular plate with insulated surfaces in 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error.If the temperature along the short edge y=0 is given by $u(x,0) = 20x, 0 \leq x \leq 5\ and\ u(x,0) = 20(10-x), 5 \leq x \leq 10$ while the two long edges x=0 and x=10 as well as the other short edge are kept at 0^0c.Find the temperature function u(x,y) in steady state at any point of the plate.

2. A square plate is bounded by the lines $\boldsymbol{x} = \boldsymbol{0}$, $\boldsymbol{x} = \boldsymbol{a\ and\ y} = \boldsymbol{0}$, $\boldsymbol{y} = \boldsymbol{b}$. Its surfaces are insulated and the temperature along $y = b$ is kept at 100^0c. While the temperature along other three edges are at 0^0c. Find the steady state temperature at any point in the plate. **(N/D-14)**

3. A square plate is bounded by the lines $\boldsymbol{x} = \boldsymbol{0}$, $\boldsymbol{x} = \boldsymbol{20\ and\ y} = \boldsymbol{0}$, $\boldsymbol{y} = \boldsymbol{20}$. Its faces are insulated. The temperature along the upper horizontal edge is given by $(\boldsymbol{x}, \boldsymbol{20})(\boldsymbol{20} - \boldsymbol{x})$, $\boldsymbol{0} <$ x < 20, while the other three edge are kept at 0^0c. Find the steady state temperature distribution (x, y) in the plate. **(N/D-16)**

3. Along rectangular piate with insulated surface is l cm wide. If the temperature along one short edge i $(\boldsymbol{x}, \boldsymbol{0}) = (\boldsymbol{lx} - \boldsymbol{x^2})$ $\boldsymbol{0} < x < l$ s, while the other two long edges $= 0\ and\ x = 1$ as well as the other short edge are kept at 0^0c, find the steady state temperature function $u(x, y)$

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